4-1. The smaller the standard deviation, the greater the precision. There is no necessary relationship between standard deviation and accuracy. The statistics that we do in this chapter pertains to precision, not accuracy.

4-2. (a) $\mu \pm \sigma$ corresponds to $z = -1$ to $z = +1$. The area from $z = 0$ to $z = +1$ is 0.341 3. The area from $z = 0$ to $z = -1$ is also 0.341 3.

Total area (= fraction of population) from $z = -1$ to $z = +1$ = 0.682 6.

(b) $z = -2$ to $z = +2$ $\Rightarrow$ area = $2 \times 0.477\,3$ = 0.954 6

(c) $z = 0$ to $z = +1$ $\Rightarrow$ area = 0.341 3

(d) $z = 0$ to $z = 0.5$ $\Rightarrow$ area = 0.191 5

(e) Area from $z = -1$ to $z = 0$ is 0.341 3. Area from $z = -0.5$ to $z = 0$ is 0.191 5. Area from $z = -1$ to $z = -0.5$ is 0.341 3 – 0.191 5 = 0.149 8.

4-3. (a) Mean $= \frac{1}{8}(1.526\,60 + 1.529\,74 + 1.525\,92 + 1.527\,31 + 1.528\,94 +$

$1.528\,04 + 1.526\,85 + 1.527\,93) = 1.527\,67$

(b) Standard deviation =

$$\sqrt{\frac{(1.526\,60 - 1.527\,67)^2 + \cdots + (1.527\,93 - 1.527\,67)^2}{8 - 1}} = 0.001\,26$$

(c) Variance = $(0.001\,26)^2 = 1.59 \times 10^{-6}$

4-4. (a) 1000 hours corresponds to $z = (1000 - 845.2)/94.2 = 1.643$.

To find the area from $\bar{x}$ to $z = 1.643$, we interpolate between $z = 1.6$ and $z = 1.7$. The area from $\bar{x}$ to $z = 1.6$ in Table 4-1 is 0.445 2 and the area from $\bar{x}$ to $z = 1.7$ is 0.455 4.

Area between $z = 1.6$ and $z = 1.643$

$$= \underbrace{\left(\frac{1.643 - 1.600}{1.700 - 1.600}\right)}_{\substack{\text{Fraction of} \\ \text{interval between} \\ z = 0.1.6 \text{ and } z = 1.7}} \underbrace{(0.455\,4 - 0.445\,2)}_{\substack{\text{Area between} \\ z = 1.6 \text{ and } z = 1.7}} = 0.004\,4$$

Area from $\bar{x}$ to $z = 1.643$ = (area from $\bar{x}$ to $z = 1.6$ + area from $z = 1.6$ to $z = 1.643$) = 0.445 2 + 0.004 4 = 0.449 6

Area beyond $z = 1.643$ is 0.500 0 – 0.449 6 = 0.050 4

(b) 800 to 845.2: $z = -0.479\,8 \Rightarrow$ area = 0.184 2

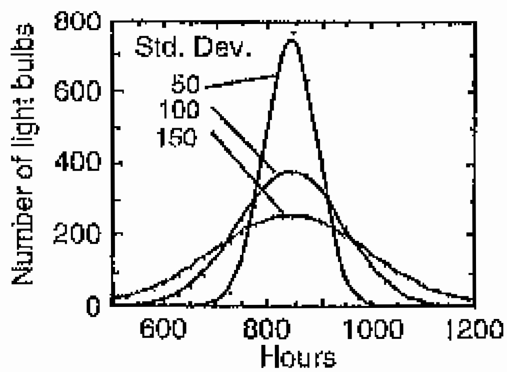
845.2 to 900: $z = 0.5817 \Rightarrow$ area $= 0.2195$

Total area from 800 to 900 $= 0.4037$

4-5. The values 14.55 to 14.60 correspond to the range ($z = 0.5047$) to ($z = 0.9720$). Interpolating in Table 4-1, the area between the two is $0.3342 - 0.1931 = 0.1411$.

4-6. Your curve should look like the one in Figure 4-1.

4-7. Use the same spreadsheet as in the previous problem, but vary the standard deviation. Here are the results:



4-8. A confidence interval is a region around the measured mean in which the true mean is likely to lie.

4-9. Since the bars are drawn at a 50% confidence level, 50% of them ought to include the mean value if many experiments are performed. 90% of the 90% confidence bars must reach the mean value if we do enough experiments. The 90% bars must be longer than the 50% bars because more of the 90% bars must reach the mean.

4-10. Case 1: Comparing a measured result to a "known" value. (Use Equation 4-7.)
Case 2: Comparing replicate measurements. (Use Equations 4-8 and 4-9 if the two standard deviations are not significantly different from each other. Use Equations 4-8a and 4-9a if the standard deviations are significantly different.)
Case 3: Comparing individual differences. (Use Equations 4-10 and 4-11.)

4-11. $\bar{x} = 0.14_8$, $s = 0.03_4$

90% confidence: $\mu = 0.14_8 \pm \dfrac{(2.015)(0.03_4)}{\sqrt{6}} = 0.14_8 \pm 0.02_8$

99% confidence: $\mu = 0.14_8 \pm \dfrac{(4.032)(0.03_4)}{\sqrt{6}} = 0.14_8 \pm 0.05_6$

4-12. 99% confidence interval: $\bar{x} \pm \dfrac{(3.707)(0.000\,07)}{\sqrt{7}} = \bar{x} \pm 0.000\,10$

(1.527 83 to 1.528 03).

4-13. (a) dL = deciliter = 0.1 L = 100 mL

(b) $F_{calculated} = (0.05_3/0.04_2)^2 = 1.5_9 < F_{table} = 6.26$ (for 5 degrees of freedom in the numerator and 4 degrees of freedom in the denominator). Since $F_{calculated} < F_{table}$, we can use the following equations:

$$s_{pooled} = \sqrt{\frac{0.53^2(5) + 0.42^2(4)}{6+5-2}} = 0.48_4$$

$$t = \frac{|14.5_7 - 13.9_5|}{0.48_4}\sqrt{\frac{6 \cdot 5}{6+5}} = 2.12 < 2.262$$ (listed for 95% confidence and 9 degrees of freedom). The results agree and the trainee should be released.

4-14.

Sample	Method 1	Method 2	d_i	$d_i - \bar{d}$	$(d_i - \bar{d})^2$
A	0.013 4	0.013 5	−0.000 1	+0.000 6	3.6×10^{-7}
B	0.014 4	0.015 6	−0.001 2	−0.000 5	2.5×10^{-7}
C	0.012 6	0.013 7	−0.001 1	−0.000 4	1.6×10^{-7}
D	0.012 5	0.013 7	−0.001 2	−0.000 5	2.5×10^{-7}
E	0.013 7	0.013 6	+0.000 1	+0.000 8	6.4×10^{-7}
			$\bar{d}$ = −0.000 70		sum = 16.6×10^{-7}

$$s_d = \sqrt{\frac{\Sigma(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{16.6 \times 10^{-7}}{4}} = 6.4 \times 10^{-4}$$

$$t = \frac{0.000\,70}{0.000\,64}\sqrt{5} = 2.4_3 < 2.776$$ (Student's t for 4 degrees of freedom)

The difference is not significant.

4-15. In the spreadsheet below we find $t_{calculated}$ (which is labeled t Stat in cell F9) is less than t_{table} (t Critical two-tail in cell F13). Therefore the difference is *not* significant.

	A	B	C	D	E	F	G
1	Comparing Individual Differences				t-Test: Paired Two Sample for Means		
2						*Variable 1*	*Variable 2*
3	Sample	Method A	Method B		Mean	1.995	1.935
4	1	1.46	1.42		Variance	0.38515	0.36699
5	2	2.22	2.38		Observations	6	6
6	3	2.84	2.67		Pearson Correlation	0.980343	
7	4	1.97	1.80		Hypothesized Mean Diff	0	
8	5	1.13	1.09		df	5	
9	6	2.35	2.25		t Stat	1.2	
10					P(T<=t) one-tail	0.141946	
11	t Stat in cell F9 is less than				t Critical one-tail	2.015049	
12	t Critical in cell F13, so the				P(T<=t) two-tail	0.283891	
13	difference is not significant				t Critical two-tail	2.570578	

4-16. In cell G9, $t_{calculated} = 0.109\,94$. For = 10 − 1 = 9 degrees of freedom, t_{table} in Table 4-2 for 95% confidence is 2.262, which agrees with the value in cell G13. Since $t_{calculated} < t_{table}$, the difference between the two methods is not significant.

	A	B	C	D	E	F	G	H
1	Comparing individual differences					t-Test: Paired Two Sample for Means		
2							*Variable 1*	*Variable 2*
3	Sample	Gravimet	Spectro			Mean	32.06	32
4	1	25.5	24.4			Variance	288.3671	273.06667
5	2	9.2	10.0			Observations	10	10
6	3	26.2	25.8			Pearson Correlation	0.995065	
7	4	50.5	47.3			Hypothesized Mean Differ	0	
8	5	25.6	28.6			df	9	
9	6	16.7	15.0			t Stat	0.109944	
10	7	42.9	43.2			P(T<=t) one-tail	0.457433	
11	8	56.0	54.9			t Critical one-tail	1.833114	
12	9	53.5	53.7			P(T<=t) two-tail	0.914866	
13	10	15.5	17.1			t Critical two-tail	2.262159	

4-17. $\mu = \bar{x} \pm \frac{(2.353)(1\%)}{\sqrt{4}} = \bar{x} \pm 1.1_8\ \% < 1.2\%$. The answer is yes.

4-18. For indicators 1 and 2: $F_{calculated} = (0.002\ 25/0.000\ 98)^2 = 5.2_7 > F_{table} \approx 2.2$ (for 27 degrees of freedom in the numerator and 17 degrees of freedom in the denominator). Since $F_{calculated} > F_{table}$, we use the following equations:

$$\text{Degrees of freedom} = \left\{ \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\left(\frac{(s_1^2/n_1)^2}{n_1+1} + \frac{(s_2^2/n_2)^2}{n_2+1}\right)} \right\} - 2$$

$$= \left\{ \frac{(0.002\ 25^2/28 + 0.000\ 98^2/18)^2}{\left(\frac{(0.002\ 25^2/28)^2}{28+1} + \frac{(0.000\ 98^2/18)^2}{18+1}\right)} \right\} - 2 \approx 40.9 \approx 41$$

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{|0.095\ 65 - 0.086\ 86|}{\sqrt{0.002\ 25^2/28 + 0.000\ 98^2/18}} = 18.2$$

This is much greater than t for 41 degrees of freedom, which is ~2.02. The difference is significant.

For indicators 2 and 3: $F_{calculated} = (0.001\ 13/0.000\ 98)^2 = 1.3_3 < F_{table} \approx 2.2$ (for 28 degrees of freedom in the numerator and 17 degrees of freedom in the denominator). Since $F_{calculated} < F_{table}$, we use the following equations:

$$s_{pooled} = \sqrt{\frac{s_1^2\ (n_1-1) + s_2^2\ (n_2-1)}{n_1 + n_2 - 2}} = 0.001\ 075\ 8$$

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{pooled}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} = 1.39 < 2.02 \Rightarrow$$ difference is not significant.

4-19. $$s_{pooled} = \sqrt{\frac{30.0^2(31) + 29.8^2(31)}{32 + 32 - 2}} = 29.9$$

$t = \frac{52.9 - 31.4}{29.9} \sqrt{\frac{32 \cdot 32}{32 + 32}} = 2.88$. The table gives t for 60 degrees of freedom, which is close to 62. The difference is significant at the 95 and 99% levels.

4-20. $\bar{x} = 97.0_0$, $s = 1.65_5$

$$t_{\text{calculated}} = \frac{|\text{known value} - \bar{x}|}{s}\sqrt{n} = \frac{|94.6 - 97.0_0|}{1.66}\sqrt{5} = 3.2_3$$

For 4 degrees of freedom and 95% confidence, $t_{\text{table}} = 2.776$.

Because $t_{\text{calculated}}$ (3.23) > t_{table} (2.776), the difference <u>is</u> significant.

With one more measurement of 94.5, $\bar{x} = 96.5_8$, $s = 1.8_0$, and $t_{\text{calculated}}$ (2.6_9) > t_{table} (2.571). The difference <u>is</u> still significant.

4-21. (a) Rainwater:

$F_{\text{calculated}} = (0.008/0.005)^2 = 2.5_6 < F_{\text{table}} = 4.53$ (for 4 degrees of freedom in the numerator and 6 degrees of freedom in the denominator). Since $F_{\text{calculated}} < F_{\text{table}}$, we use the following equations:

$$s_{\text{pooled}} = \sqrt{\frac{0.005^2(6) + 0.008^2(4)}{7 + 5 - 2}} = 0.006_{37}$$

$$t_{\text{calculated}} = \frac{0.069 - 0.063}{0.006_{37}}\sqrt{\frac{7 \cdot 5}{7 + 5}} = 1.6_1 < t_{\text{table}} = 2.228.$$

Difference is <u>not</u> significant.

Drinking water:

$F_{\text{calculated}} = (0.008/0.007)^2 = 1.3_1 < F_{\text{table}} = 6.39$ (for 4 degrees of freedom in the numerator and 4 degrees of freedom in the denominator). Since $F_{\text{calculated}} < F_{\text{table}}$, we use the following equations:

$$s_{\text{pooled}} = \sqrt{\frac{0.007^2(4) + 0.008^2(4)}{5 + 5 - 2}} = 0.007_{52}$$

$$t = \frac{0.087 - 0.078}{0.007_{52}}\sqrt{\frac{5 \cdot 5}{5 + 5}} = 1.8_9 < 2.306.$$ Difference is <u>not</u> significant.

(b) Gas chromatography:

$$s_{\text{pooled}} = \sqrt{\frac{0.005^2(6) + 0.007^2(4)}{7 + 5 - 2}} = 0.005_{88}$$

$$t = \frac{0.078 - 0.069}{0.005_{88}}\sqrt{\frac{7 \cdot 5}{7 + 5}} = 2.6_1 > 2.228.$$ Difference <u>is</u> significant.

Spectrophotometry:

$$s_{\text{pooled}} = \sqrt{\frac{0.008^2(4) + 0.008^2(4)}{5 + 5 - 2}} = 0.008_{00}$$

$$t = \frac{0.087 - 0.063}{0.008_{00}}\sqrt{\frac{5 \cdot 5}{5 + 5}} = 4.7_4 > 2.306.$$ Difference <u>is</u> significant.

4-22. $Q = (216 - 204) / (216 - 192) = 0.50 < 0.64$. Retain 216.